

3418

**FINAL TECHNICAL REPORT**

Name:

INSTITUTO DE PRODUCTOS LACTEOS Y  
DERIVADOS GRASOS (formerly Depar-  
tamento de Lipoquimica).- Patro-  
nato "Juan de la Cierva".- Juan  
de la Cierva, 3, Madrid-6 (Spain)

Director: Dr. D. Martín

Pr. Investigator: Dr. D. Martín

Project title:

DETERMINATION OF THE THERMAL AND  
RELATED PHYSICAL PROPERTIES OF MILK  
AND MILK PRODUCTS, TO PROVIDE BASIC  
INFORMATION NECESSARY FOR THE PREPA-  
RATION OF IMPROVED MILK CONCENTRA-  
TES.

Project No.:

UR - E25 - (60) - 37

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**Project No.:**

UR - E25 - (69) - 37

**Grant No.:**

FG - Sp - 140

This project consists of two parts:

A - Literature Review

B - Experimental work

The literature review covers the majority of the papers so far devoted to physicochemical properties on dairy products; such reviewed properties are: density, vapour pressure, surface tension, specific heat, heat of vaporization, heat of fusion, thermal expansion, thermal conductivity, emissivity, dielectric constant and rheological properties.

The experimental work deals with thermal properties determinations on skimmilk, half-half milk, whole milk and their concentrates up to about 50% total solids and 20, 30 & 40% fat creams, over a temperature range between 0° and 80°C. The measured thermal properties are the following:

I - Thermal Expansion

II - Viscosity

III - Thermal Conductivity

IV - Heat Capacity

Every property is treated in a separated section just numbered as above, independently paged and comprising the following paragraphs:

- 1.- Introduction
- 2.- Method
- 3.- Results
- 4.- Discussion
- 5.- Conclusion
- 6.- Bibliography
- 7.- Need for Additional Research
- 8.- List of publications
- 9.- Appendix

Aiming to facilitate the study of this manuscript, every section has been written in full detail, no matter if samples' preparation, chemical analysis methods, results, etc., were common to two or more sections. Following this idea, tables and figures have been accordingly numbered and included in their respective section.

A) L I T E R A T U R E      R E V I E W

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A.- Literature Review.

Bibliographic cards have been prepared and forwarded to A.R.S. throughout the contract, according to the following:

M a t t e r	Code	1 <sup>st</sup> AR	2 <sup>nd</sup> RPR	2 <sup>nd</sup> AR	3 <sup>rd</sup> RPR	3 <sup>rd</sup> AR	4 <sup>th</sup> RPR	4 <sup>th</sup> AR	5 <sup>th</sup> RPR	FR	Total
Density	D	116	15	8	1	1	2	5	2	-	150
Vapour pressure	VP	4	4	--	--	--	--	--	--	-	8
Surface tension	ST	68	4	5	--	--	--	--	--	1	78
Specific heat	SH	18	3	7	2	--	5	1	1	-	37
Heat of vaporization.	HV	--	--	--	--	--	--	--	--	-	--
Heat of fusion	HF	2	2	--	1	--	--	--	--	-	5
Thermal expansion.	TE	18	10	8	--	1	6	--	1	-	44
Thermal conductivity.	TC	25	2	1	2	1	2	1	--	1	35
Emissivity	E	--	--	--	--	--	--	--	--	-	--
Dielectric constant.	DC	3	6	8	3	--	--	--	2	-	22
Rheology	R	145	94	74	13	4	6	9	2	11	358
Total		399	140	111	22	7	21	18	8	13	737

B) EXPERIMENTAL WORK

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I - THERMAL EXPANSION

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## I - THERMAL EXPANSION

### 1.- Introduction

Almost all the papers concerning dilatometry are devoted to butterfat in relation with its crystallization - kinetic, solid fat index measurements and the rheological properties of butter. Many other papers have been also published in connection with specific gravity or density of milk but the general point of view was the relation between lactodensimeter reading and chemical composition of milk.

The following papers are the most related ones with the purpose of this work:

- Settimj<sup>1</sup> (1925) studied dilatation coefficients in whole milk reporting the following ones:

Whole milk ( $\rho_{15} = 1.031$ )	$0^\circ - 5^\circ\text{C} \rightarrow V_t = V_o (1 + 0.0001t)$
	$5^\circ - 10^\circ\text{C} \rightarrow V_t = V_o (0.9999 + 0.00012t)$
	$10^\circ - 30^\circ\text{C} \rightarrow V_t = V_o (0.9997 + 0.0001t + 0.000004t^2)$

- Chekulaeva<sup>2</sup> (1953) carried out density measurements on creams between 25 and 45% fat content and from 5 to 85 C, giving two expressions, i.e., one for correlating density with temperature at constant fat content and another for correlating density with fat content at constant temperature:

$$\rho_z = \rho_x - 0.000534 (t - x) \quad \text{g/ml.}$$

$$\rho_f = \rho^* - 0.00131 (f - f^*)$$

- Babad and coworkers<sup>3</sup> (1954) established an equation relating lactometer reading with total solids and temperature on skim milks:

$$L = (4.259 - 0.00113t - 0.000050t^2)s - (1.249 + 0.0282t + 0.0046t^2)$$

- Short<sup>4</sup> (1955), measuring density on several whole milk samples, quoted equations for them and also for partial contributions of 1% fat and 1% SNF:

$$1\% \text{ fat} \rightarrow \rho - 1 = 4.8 - 0.39t + 0.0061t^2 - 0.00002t^3$$

$$1\% \text{ SNF} \rightarrow \rho - 1 = 3.8 - 0.08t - 0.010t^2 + 0.00004t^3$$

- Rutz et al.<sup>5</sup> (1955) reported 0.00038 as a temperature coefficient to convert milk density at any temperature between 18 and 44 C.
- Watson and Tittsler<sup>6</sup> (1961) made density measurements on milk over 1 to 10 C temperature range and reporte several expressions relating density with temperature and composition; the most general one being in function of temperature, fat and solid non-fat contents:

$$\rho = 1.003073 - 0.000179t - 0.000368f + 0.003744m$$

- Tverdokhleb and Avvakumov<sup>7</sup> (1962) reported thermal expansion coefficients for skimmilk (0.005-0.01%f) as follows:

0.000031	ml/g/°C	for	0° to 5°C zone
0.000341	"	for	35° to 40°C zone

- Verma and Garg<sup>8</sup> (1965) determined volume increases (1°C step between 4° and 40°C, 5°C step between 40° and 95°C)

on skimmilk (0.1%f) and cow's milk (1 to 5%f) and computed average expansion coefficients (original paper not seen).

- Andrianov and Tverdokhleb<sup>9</sup> (1967) and Andrianov, Tverdokhleb and Makarova<sup>10</sup> (1968) studied creams density and recommend additive formulas for practical purposes. In the last paper the authors reported expression for computing density at a temperature when  $\rho_{40}$  is known:

$$\rho_t = \rho_{40} + \alpha(t-40) + \beta(t-40)$$

where t can take any value between 25° and 90°C.  $\alpha$  and  $\beta$  parameters are tabulated for creams between 28.5 and 83.4% fat content. An equation is also given relating  $\rho_t$  with fat content and temperature:

$$\rho_t = 1.0435 - 1.17 \times 10^{-3} f - (0.520 \times 10^{-3} + 1.60 \times 10^{-6} f)t \text{ g/cm}^3$$

where t ranges from 40° to 90°C and f content from 30 to 83%.

So that, bibliographic data is rather scarce and really, no attempt has been made in order to relate differential or integral dilations with temperature and composition on dairy products.

## 2.- Method

The dilatometers, volumetric type, were built in Pyrex glass and featured a bulb of approx. 10 cubic centimeters, a capillary of 1 mm. Ø and 40 cm. of graduated scale.

The bulb was coarsely gauged for density calculations, and the capillary was previously calibrated by the method of the "mercury drop". The correct performance of the dilatometers was verified by the measurement of distilled water.

The dilatometers were totally immersed into the thermostatization liquid so as to prevent corrections due to emergent stem. In order to perform the lectures of the capillary column a thermostatic bath with viewing port was constructed. The thermostatization obtained being better than  $0.05^{\circ}\text{C}$ .

Mercury was used as confining liquid, reason for which the experimental data were adequately corrected. Proper corrections were also made in connection with glass expansion, assuming an average cubic expansion coefficient of  $10^{-5}^{\circ}\text{C}^{-1}$  for Pyrex glass.

The static method was followed so that the lecture of the mercury level in the capillary was made once the completion of the equilibrium had taken place. Lectures were made by  $5^{\circ}\text{C}$  steps, this being sufficient to establish quite well the respective differential curve.

### 3.- Results

Thermal expansions were measured between 0° and 80°C on the following samples:

- Skimmilk and its concentrates up to 40% total solids
- Half-Half milk and its concentrates up to 40% total solids.
- Whole milk and its concentrates up to 50% total solids.
- Creams at 20, 30 and 40% fat content.

All them were analyzed for fat content (f) according to Gerber method<sup>11</sup>, total solids content (s) by desecation<sup>12</sup>, nitrogen content (N) by Kjeldahl<sup>13</sup>, homogenization efficiency by Farral Index<sup>14</sup> and, in some cases, for reduction time (RT)<sup>15</sup> and pH at 25°C. Results are given in tables I-1 to I-4

T A B L E      I-1  
 Chemical Analysis Results: Skimmilks serie.

Code	Sample No.	% fat	% total solids.	Mean Values	Observations
S-1	12	0.07	8.14	%f = 0.12	- Leche desnata da (desgrasada)
	13	0.12	8.26	%s = 8.18	
	14	0.12	8.15	%N = 0.45	- Commercial
	15	0.17	8.16	FI = --- pH = 6.5 RT > 6 h.	- Sterilized
S'-1	5	0.60	10.01	%f = 0.60	- Leche parcial mente desnata da.
	6	0.60	10.10	%s = 10.05	
				%N = 0.53	- Commercial
					- Sterilized
S-2.03	21	0.07	16.30	%f = 0.18	
	24	0.17	16.57	%s = 16.62	
	26	0.19	16.81	%N = 0.91	
	27	0.18	16.46	FI = ---	
	28	0.18	16.98	pH = 6.2 RT > 6 h.	
S-3.46	124	0.36	28.50	%f = 0.35	
	126	0.35	28.20	%s = 28.31	
	127	0.32	28.01	%N = 1.50	
	128	0.37	28.56	FI = ---	
	129	0.36	28.30	pH = 6.6 RT > 7 h.	
S-4.58	130	0.56	37.53	%f = 0.45	
	131	0.58	38.28	%s = 37.5	
	132	0.36	37.27	%N = 1.97	
	133	0.38	37.40	FI = ---	
	135	0.35	37.00	pH = 6.5 RT > 6 h.	

T A B L E I-2

Chemical Analysis Results: Half-Half milks serie

Code	Sample No.	% fat	% total solids.	Mean Values	Observations
H-1	87	1.64	9.22	%f = 1.56	- Leche parcialmen te desnatada al 1.5%
	88	1.63	9.15	%s = 9.17	- Commercial
	89	1.52	9.17	%N = 0.44	- Sterilized
	90	1.50	9.15	FI = 0-10	- Homogenized
	91	1.50	9.18	pH = 6.2 RT > 6 h.	
H-2.02	92	3.01	18.67	%f = 3.05	- Laboratory prepa red from H-1
	93	2.99	17.77	%s = 18.53	
	95	3.15	18.93	%N = 0.90	
	96	3.01	18.64	FI = 5-10	
	97	3.07	18.65	pH = 6.1 RT > 6 h.	
H-3.17	102	4.78	28.67	%f = 4.87	- Laboratory prepa red from H-1
	108	4.89	29.04	%s = 29.07	
	114	4.83	29.63	%N = 1.4	
	115	5.10	29.11	FI = 5-10	
	117	4.79	18.59	pH = 6.5	
	118	4.90	28.85	RT > 6 h.	
	123	4.85	29.64		
H-4.25	150	5.63	38.34	%f = 5.6	- Laboratory prepa red from H-1
	156	5.60	39.20	%s = 39.0	
	159	5.60	39.30	%N = 1.9	- See Appendix
	160	5.60	39.02	FI > 10 pH = 6.4 RT > 5 h.	

T A B L E      I-3  
 Chemical Analysis Results: Whole milks serie

Code	Sample No.	% fat	% total solids.	Mean Values	Observations
W-1	41	3.06	11.25	%f = 3.05	- Leche entera
	42	3.09	11.09	%s = 11.03	- Commercial
	42-bis	3.09	11.09	%N = 0.46	- Sterilized
	43	3.02	10.86	FI = 0	- Homogenized
	44	3.01	10.88	pH = 6.5 RT > 6 h.	
W-1.49	50	4.63	16.47	%f = 4.58	
	51	4.50	16.19	%s = 16.43	
	52	4.58	16.47	%N = 0.68	- Laboratory prepared from W-1 and W-3
	53	4.60	16.59	FI = 0-5 pH = 6.5 RT > 6 h.	
W-1.99	45	5.97	21.80	%f = 6.00	- Leche concentrada 1:2
	46	5.99	21.92	%s = 21.92	- Commercial
	47	6.09	22.40	%N = 0.92	- Sterilized
	48	5.97	21.80	FI = 0	- Homogenized
	49	5.97	21.70	pH = 6.3 RT > 6 h.	
W-2.49	54	7.31	27.42	%f = 7.28	- Leche evaporada
	55	7.31	27.65	%s = 27.47	- Commercial
	56	7.36	27.74	%N = 1.18	- Sterilized
	57	7.21	27.56	FI = 0	- Homogenized
	58	7.21	26.97	pH = 6.2 RT > 6 h.	
W-3.59	59	10.37	39.02	%f = 10.51	- Leche concentrada 1/4
	60	10.45	38.55	%s = 39.57	- Commercial
	61	10.65	38.81	%N = 1.65	- Pasteurized
	67	10.71	40.56	FI = 0	- Non homogenized
	69	10.62	40.20	pH = 6.3	
	70	10.38	39.40	RT > 4 h.	
	72	10.38	40.47		
W-4.48	66	13.89	48.42	%f = 13.9	
	77	13.87	49.20	%s = 49.4	
	79	13.93	49.74	%N = 1.99	- Laboratory prepared from W-3
	80	13.84	49.12	FI = 0-5	
	81	13.90	49.52	pH = 5.7	- See Appendix
	84	13.9	49.4	RT > 5 h.	
	85	13.9	49.3		
	86	13.9	49.7		

T A B L E      I-4  
 Chemical Analysis Results: creams serie

Code	Sample No.	% fat	% total solids	Mean Values	Observations
C-2	136	19.91	26.80	%f = 20.05	- Factory cream
	138	20.39	27.09	%s = 26.78	
	139	19.29	25.92	%N = 0.40	
	142	21.69	29.07	FI = --	
	143	19.43	25.92	pH = 6.4	
	145	20.60	27.02	RT > 2 h	
	147	20.05	26.65		
	148	19.22	25.97		
	150	19.86	26.57		
C-3	151	29.45	34.67	%f = 29.63	- Factory cream
	153	29.64	34.50	%s = 34.48	
	164	29.87	34.60	%N = 0.19	
	166	29.35	34.10	FI = --	
	169	29.35	33.70	pH = 6.5	
	173	29.97	35.10	RT > 2 h.	
	174	29.76	34.70		
	175	29.62	34.46		
C-4 I	177	39.55	42.00	%f = 39.79	- Factory cream
	178	39.65	45.13	%s = 44.52	
	179	39.65	43.50	%N = 0.25	
	180	39.38	45.46	FI = --	
	181	39.65	45.63	pH = 6.3	
	182	39.98	44.86	RT > 2 h.	
	183	40.78	45.74		
	184	38.11	42.66		
	185	39.65	45.28		
	187	40.17	45.60		
	190a	40.17	44.45		
	191a	39.75	44.28		
	192a	40.17	45.49		
	194a	40.37	44.67		
	197a	39.75	43.05		
	199a	39.80	44.50		

Skimmilks' and half-half milks' concentrates were prepared in the laboratory by evaporation at about 50°C and water pump vacuum by means of a rotavapor. As it could be expected

Figure 1-1 Skim milks serie

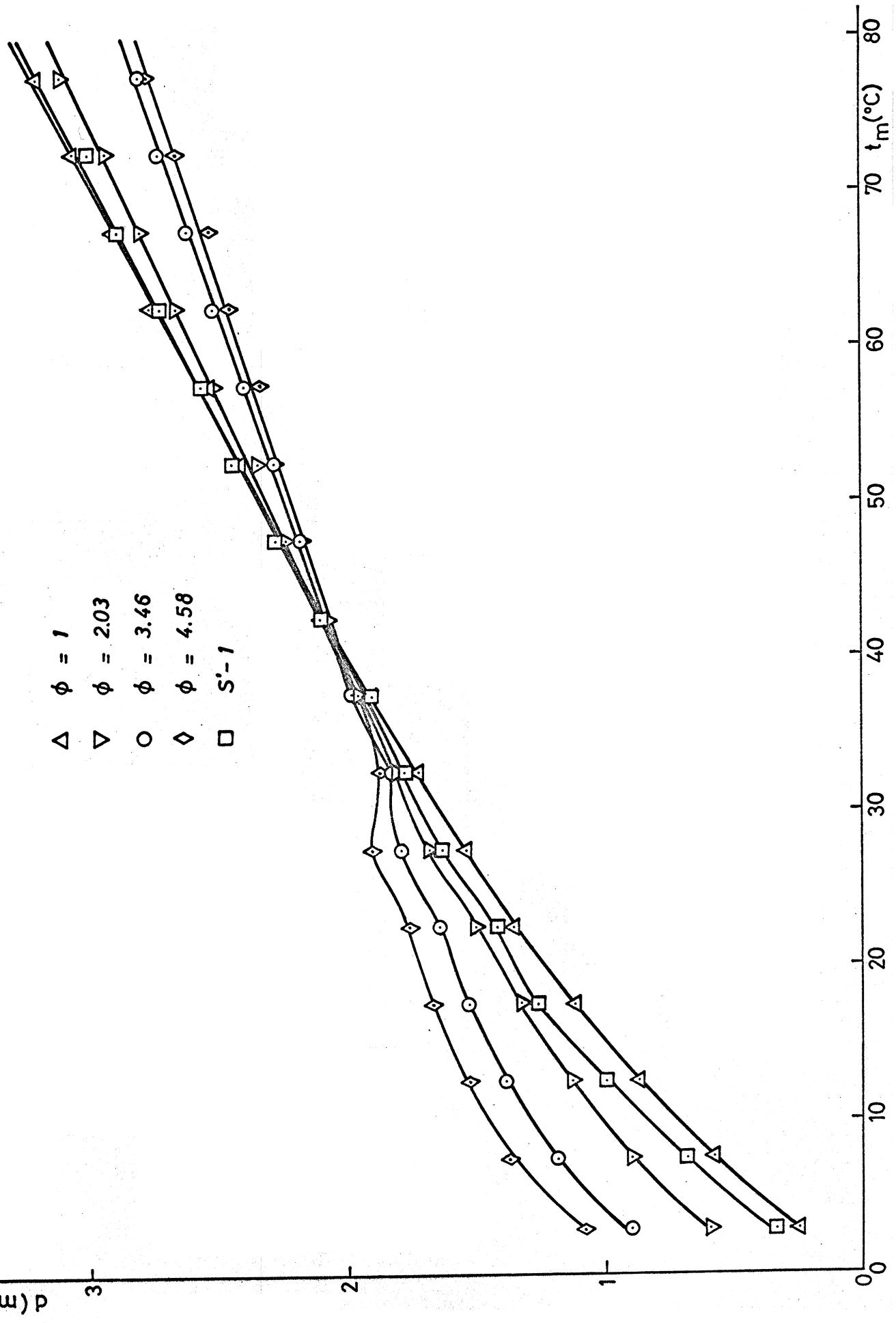


TABLE I-5.- Experimental Results in skimmilks:  $d$  ( $\text{mm}^3/\text{g}/5^\circ\text{C}$ )

Code	$t_m$ ( $^\circ\text{C}$ )	No. 12	No. 13	No. 14	No. 15	$\bar{d}$
S - 1	2.8	0.25	0.25	0.25	0.26	0.25
	7.5	0.58	0.58	0.58	0.60	0.58
	12.5	0.85	0.87	0.87	0.88	0.87
	17.5	1.11	1.13	1.14	1.12	1.12
	22.5	1.38	1.32	1.37	1.35	1.36
	27.5	1.58	1.52	1.53	1.53	1.54
	32.5	1.68	1.72	1.75	1.76	1.73
	37.5	1.88	1.87	1.91	1.92	1.90
	42.5	2.12	2.04	2.13	2.05	2.08
	47.5	2.24	2.30	2.24	2.26	2.26
	52.5	2.37	2.43	2.49	2.41	2.42
	57.5	-	2.54	2.55	2.51	2.53
	62.5	-	2.74	-	2.78	2.76
	67.5	-	2.89	-	2.91	2.90
	72.5	-	3.07	-	3.06	3.06
	77.5	-	3.20	-	3.20	3.20
Code	$t_m$ ( $^\circ\text{C}$ )	No. 21	No. 24	No. 26	No. 27	No. 28
S - 2.03	2.8	0.63	0.59	0.58	0.58	0.59
	7.5	0.87	0.89	0.95	0.87	0.90
	12.5	1.20	1.13	1.11	1.09	1.13
	17.5	1.24	1.34	1.30	1.33	1.34
	22.5	1.49	1.49	1.52	1.50	1.50
	27.5	1.67	1.69	1.70	1.68	1.67
	32.5	1.78	1.78	1.78	1.77	1.82
	37.5	1.93	1.99	1.96	1.97	1.92
	42.5	2.16	2.05	2.07	2.12	2.09
	47.5	2.19	2.28	2.23	2.26	2.23
	52.5	2.38	2.33	2.30	2.37	2.32
	57.5	2.48	2.49	2.57	2.48	2.58
	62.5	2.61	2.66	2.72	2.70	2.62
	67.5	-	2.88	2.65	2.83	2.78
	72.5	-	2.91	2.95	2.94	2.92
	77.5	-	3.10	-	-	3.10
Code	$t_m$ ( $^\circ\text{C}$ )	No. 124	No. 126	No. 127	No. 128	No. 129
S - 3.46	2.8	0.91	0.91	0.88	0.91	0.87
	7.5	1.19	1.17	1.18	1.21	1.19
	12.5	1.49	1.33	1.32	1.44	1.37
	17.5	1.50	1.45	1.62	1.58	1.51
	22.5	1.69	1.65	1.58	1.67	1.63
	27.5	1.78	1.78	1.79	1.86	1.79
	32.5	1.80	1.76	1.80	1.82	1.80
	37.5	1.98	1.92	1.97	2.05	1.93
	42.5	2.12	2.03	2.07	2.14	2.09
	47.5	2.18	2.17	2.21	2.20	2.14
	52.5	2.30	2.24	2.27	2.29	2.28
	57.5	2.38	2.39	2.40	2.37	2.42
	62.5	2.53	2.47	2.51	2.60	2.50
	67.5	2.68	2.51	2.64	2.68	2.59
	72.5	2.80	2.70	2.70	2.75	2.69
	77.5	--	-	2.84	2.77	2.78

Figure 1-2 Half-half milks serie

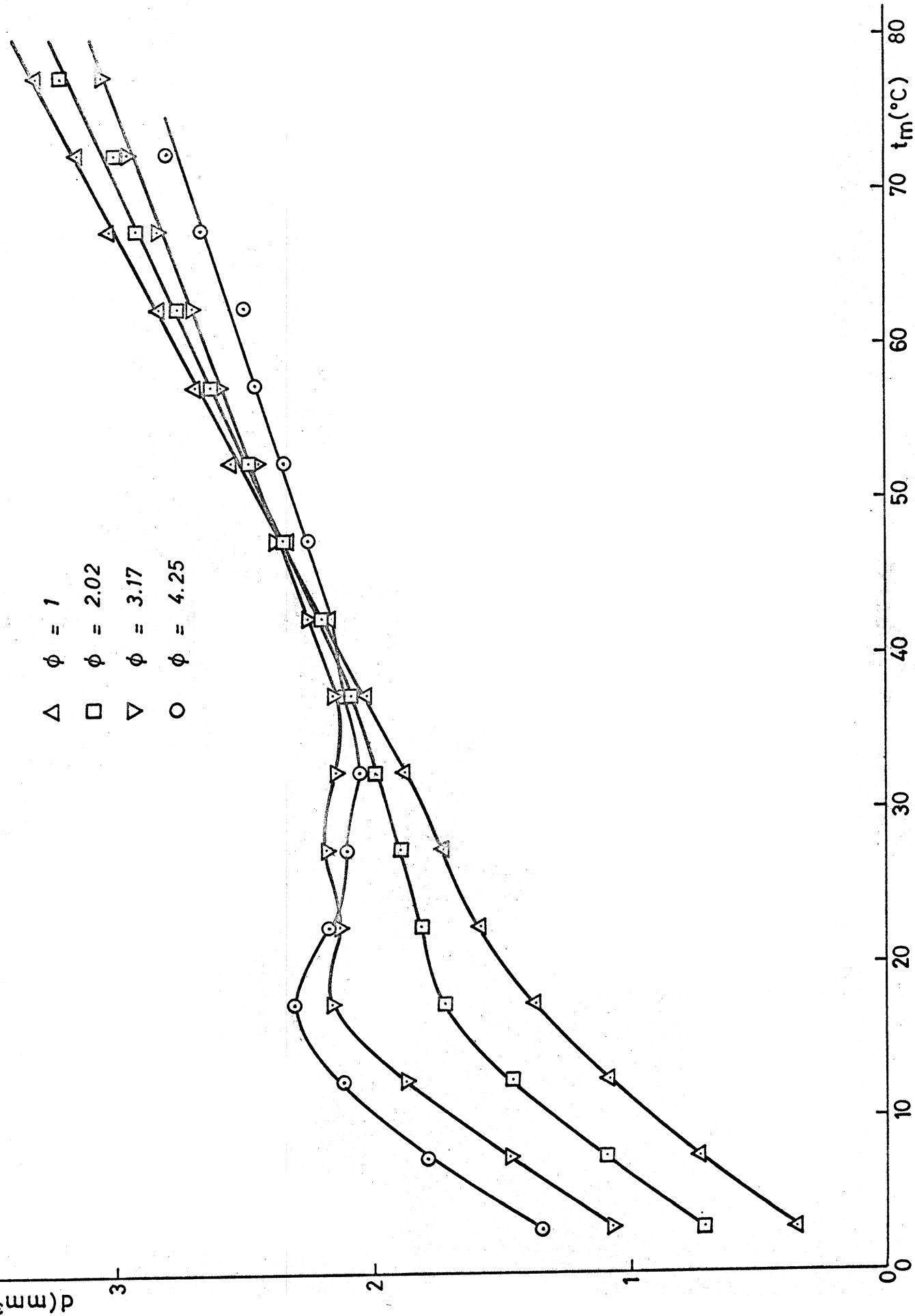


TABLE I-6.- Experimental results in half-half milks:  $d(\text{mm}^3/\text{g}/5^\circ\text{C})$

Code	$t_m (\text{ }^\circ\text{C})$	No. 87	No. 88	No. 89	No. 90	No. 91	$\bar{d}$		
H - 1	2.8	0.40	0.34	0.35	0.31	0.35	0.35		
	7.5	0.73	0.73	0.74	0.74	0.70	0.73		
	12.5	1.11	1.08	1.06	1.07	1.10	1.08		
	17.5	1.37	1.37	1.35	1.36	1.42	1.37		
	22.5	1.55	1.54	1.61	1.58	1.60	1.58		
	27.5	1.71	1.74	1.70	1.73	1.75	1.73		
	32.5	1.97	1.83	1.86	1.87	1.88	1.88		
	37.5	2.01	2.02	2.05	1.98	2.07	2.03		
	42.5	2.22	2.16	2.17	2.15	2.19	2.17		
	47.5	2.36	2.35	2.38	2.30	2.45	2.37		
	52.5	2.55	2.54	2.51	2.56	2.55	2.54		
	57.5	2.63	2.66	2.75	2.70	2.66	2.68		
	62.5	2.86	2.73	2.86	2.78	2.87	2.82		
	67.5	3.07	2.96	2.96	2.98	3.09	3.01		
	72.5	3.15	3.10	3.13	3.14	3.14	3.13		
	77.5	-	3.28	3.30	3.26	3.32	3.29		
Code	$t_m (\text{ }^\circ\text{C})$	No. 92	No. 93	No. 95	No. 96	No. 97	$\bar{d}$		
H - 2.02	2.8	0.72	0.67	0.74	0.70	0.70	0.71		
	7.5	1.07	1.10	1.11	1.	1.11	1.09		
	12.5	1.48	1.40	1.52	1.42	1.47	1.46		
	17.5	1.69	1.70	1.72	1.68	1.80	1.72		
	22.5	1.81	1.76	1.82	1.80	1.83	1.81		
	27.5	1.88	1.91	1.94	1.76	1.94	1.89		
	32.5	1.96	2.00	2.01	1.93	2.05	1.99		
	37.5	2.04	2.06	2.14	2.05	2.12	2.08		
	42.5	2.22	2.27	2.21	2.09	2.21	2.20		
	47.5	2.26	2.40	2.38	2.31	2.35	2.34		
	52.5	2.45	2.49	2.48	2.43	2.51	2.47		
	57.5	2.55	2.64	2.64	2.55	2.67	2.61		
	62.5	2.67	2.79	2.82	2.66	2.81	2.75		
	67.5	2.85	2.92	2.91	-	2.92	2.90		
	72.5	2.94	3.00	3.01	-	3.01	2.99		
	77.5	3.05	3.24	3.23	-	3.26	3.20		
Code	$t_m (\text{ }^\circ\text{C})$	No. 102	No. 108	No. 114	No. 115	No. 117	No. 118	No. 123	$\bar{d}$
H - 3.17	2.8	1.07	1.06	1.10	1.06	1.07	1.09	1.00	1.07
	7.5	1.46	1.52	1.51	1.48	1.47	1.45	1.43	1.47
	12.5	1.86	1.87	1.91	1.86	1.90	1.81	1.87	1.87
	17.5	2.04	2.15	2.19	2.17	2.25	2.17	2.15	2.16
	22.5	2.08	1.97	2.22	2.08	2.18	2.21	2.20	2.13
	27.5	2.04	2.26	2.23	2.33	2.28	2.09	2.06	2.18
	32.5	2.11	2.14	2.14	2.13	2.17	2.17	2.13	2.14
	37.5	2.16	2.17	2.13	2.21	2.15	3.17	2.11	2.16
	42.5	2.21	2.21	2.31	2.28	2.34	2.28	2.14	2.25
	47.5	2.35	2.42	2.39	2.35	2.41	2.34	2.29	2.37
	52.5	2.43	2.49	2.44	2.47	2.46	2.49	2.39	2.45
	57.5	2.58	2.56	2.59	2.66	2.57	2.62	2.55	2.59
	62.5	2.61	2.66	2.68	2.73	2.77	2.75	2.64	2.69
	67.5	2.77	2.80	2.88	2.89	2.82	2.90	2.73	2.83
	72.5	2.86	2.94	2.96	2.96	2.98	3.01	2.87	2.94
	77.5	-	-	-	-	3.11	-	2.96	3.04

Figure 1 - 3 Whole milks serie

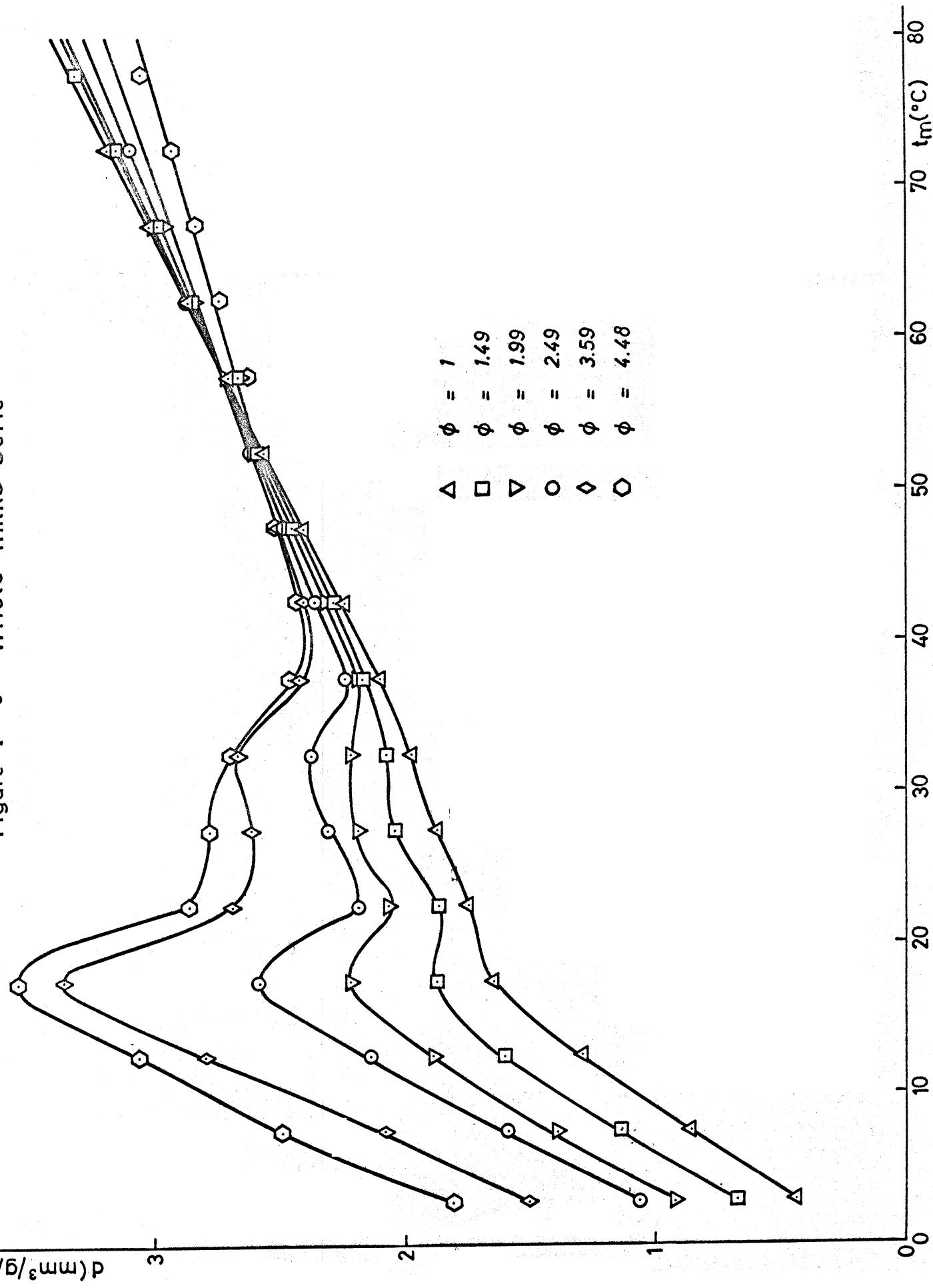


TABLE I-7.- Experimental results in whole milks:  $d$ ( $\text{mm}^3/\text{g}/5^\circ\text{C}$ ).

Code	$t_m$ ( $^\circ\text{C}$ )	No. 41	No. 42	No. 42'	No. 43	No. 44	$\bar{d}$
I W	2.8	0.44	0.46	0.45	0.43	0.41	0.44
	7.5	0.84	0.87	0.86	0.88	0.85	0.86
	12.5	1.29	1.29	1.29	1.32	1.29	1.29
	17.5	1.63	1.63	1.63	1.65	1.66	1.64
	22.5	1.75	1.71	1.77	1.79	1.70	1.75
	27.5	1.87	1.87	1.85	1.87	1.87	1.87
	32.5	1.95	1.96	1.93	2.01	2.00	1.97
	37.5	2.07	2.11	2.08	2.11	2.10	2.10
	42.5	2.20	2.25	2.22	2.26	2.25	2.24
	47.5	2.36	2.41	2.44	2.45	2.40	2.41
	52.5	2.54	2.48	2.56	2.63	2.56	2.56
	57.5	2.70	2.67	2.66	2.73	2.73	2.70
	62.5	2.83	2.86	2.85	-	2.93	2.86
	67.5	2.99	2.99	-	-	3.06	3.01
	72.5	-	3.16	-	-	3.19	3.18
	77.5	-	-	-	-	-	-
Code	$t_m$ ( $^\circ\text{C}$ )	No. 50	No. 51	No. 52	No. 53		$\bar{d}$
I W	2.8	0.65	0.67	0.68	0.69	0.67	
	7.5	1.12	1.14	1.19	1.13	1.14	
	12.5	1.56	1.56	1.65	1.62	1.60	
	17.5	1.89	1.86	1.84	1.89	1.87	
	22.5	1.86	1.84	1.85	1.88	1.86	
	27.5	2.01	2.04	2.05	2.07	2.04	
	32.5	2.07	2.04	2.10	2.08	2.07	
	37.5	2.14	2.15	2.18	2.17	2.16	
	42.5	2.25	2.28	2.31	2.24	2.27	
	47.5	2.43	2.48	2.43	2.42	2.44	
	52.5	2.52	2.53	2.60	2.56	2.55	
	57.5	2.61	2.61	2.67	2.69	2.65	
	62.5	2.82	2.83	2.87	2.85	2.84	
	67.5	2.93	2.97	3.02	2.97	2.97	
	72.5	3.07	3.14	3.16	3.14	3.14	
	77.5	-	-	3.27	3.32	3.30	
Code	$t_m$ ( $^\circ\text{C}$ )	No. 45	No. 46	No. 47	No. 48	No. 49	$\bar{d}$
I W	2.8	0.89	0.91	0.91	0.98	0.91	0.92
	7.5	1.38	1.37	1.43	1.38	1.41	1.39
	12.5	1.85	1.88	1.95	1.89	1.90	1.89
	17.5	2.25	2.24	2.22	2.18	2.15	2.21
	22.5	2.07	2.06	2.06	2.03	2.06	2.06
	27.5	2.18	2.18	2.21	2.17	2.18	2.18
	32.5	2.20	2.19	2.21	2.24	2.24	2.22
	37.5	2.20	2.15	2.19	2.16	2.19	2.17
	42.5	-	2.31	2.31	2.32	2.30	2.31
	47.5	-	2.43	2.46	2.39	2.40	2.42
	52.5	-	2.58	2.58	2.58	2.55	2.58
	57.5	-	2.68	2.68	2.69	2.70	2.69
	62.5	-	-	2.85	2.86	2.82	2.84
	67.5	-	-	-	3.01	3.00	3.01
	72.5	-	-	-	3.12	-	3.12
	77.5	-	-	-	-	-	-

**T A B L E I-7 (Continuation)**

Experimental results in whole milks:  $d(\text{mm}^3/\text{g}/5^\circ\text{C})$

TABLE I-8  
Experimental results in creams:  $d$  ( $\text{mm}^3/\text{g}/5^\circ\text{C}$ )

Code	$t_m$ ( $^\circ\text{C}$ )	No. 136	No. 138	No. 139	No. 142	No. 143	No. 145	No. 147	No. 148	No. 149	No. 150	$\bar{d}$
S - C	2.8	1.25	1.16	1.21	1.54	1.28	1.29	1.27	1.27	-	-	1.28
	7.5	2.17	2.21	2.16	2.24	2.20	2.08	2.13	2.19	-	-	2.17
	12.5	3.50	3.45	3.53	3.97	3.22	3.19	3.45	3.42	-	-	3.47
	17.5	4.32	4.00	4.98	3.56	4.13	3.84	3.88	4.01	-	-	4.09
	22.5	3.69	3.62	3.26	3.34	3.10	2.84	2.95	3.14	-	-	3.24
	27.5	3.31	3.10	3.16	3.03	2.93	2.95	3.05	3.08	-	-	3.08
	32.5	3.71	2.84	3.08	3.03	2.84	2.47	2.46	2.72	-	-	2.89
	37.5	2.82	2.52	2.68	2.67	2.64	2.52	2.68	2.61	-	-	2.64
	42.5	2.56	2.50	2.68	2.58	2.56	2.17	2.41	2.70	-	-	2.52
	47.5	2.67	2.52	2.80	2.70	2.63	2.52	2.45	2.74	-	-	2.63
S - C	52.5	2.77	2.64	2.92	2.75	2.83	2.49	2.62	2.84	2.77	2.74	2.74
	57.5	2.94	-	3.13	2.75	3.07	2.71	2.91	2.89	2.89	2.91	2.91
	62.5	3.09	-	-	-	2.79	-	-	3.09	2.99	2.99	2.99
	67.5	3.24	-	-	-	-	-	-	-	3.13	3.18	3.18
	72.5	-	-	-	-	-	-	-	-	3.25	3.25	3.25
	77.5	-	-	-	-	-	-	-	-	3.32	3.32	3.32
	2.8	1.85	1.75	-	1.82	-	-	-	-	-	-	1.81
	7.5	3.01	2.75	-	3.10	3.08	-	-	-	-	-	2.99
	12.5	4.77	4.67	4.38	4.79	4.94	-	-	-	-	-	4.71
	17.5	5.62	5.72	5.23	4.41	4.65	-	-	-	-	-	5.13
S - C	22.5	3.84	3.77	3.69	3.66	3.83	-	-	-	-	-	3.76
	27.5	3.52	3.62	3.52	3.47	3.59	-	-	-	-	-	3.54
	32.5	3.51	3.43	2.96	3.17	2.94	-	-	-	-	-	3.20
	37.5	2.98	2.82	2.78	2.74	2.78	-	-	-	-	-	2.82
	42.5	2.78	2.82	2.68	2.75	2.73	-	-	-	-	-	2.77
	47.5	2.94	2.99	2.88	2.90	2.93	-	-	-	-	-	2.92
	52.5	-	2.89	2.89	2.95	3.00	2.85	2.93	2.98	2.93	2.95	2.95
	57.5	-	-	3.06	3.09	3.16	3.13	3.12	3.13	3.10	3.10	3.10
	62.5	-	-	3.11	3.21	3.20	3.16	3.34	3.19	3.20	3.20	3.20
	67.5	-	-	-	-	-	3.22	3.34	3.47	3.45	3.44	3.44
S - C	72.5	-	-	-	-	-	3.40	3.54	3.56	3.54	3.53	3.53
	77.5	-	-	-	-	-	3.50	-	-	-	-	-

T A B L E I-8      (Continuation)

TABLE I - 9

Relative Integral Dilatation: D( $\text{mm}^3/\text{g}$ )

t ( $^{\circ}\text{C}$ )	S-1	S-2.03	S-3.46	H-1	H-2.02	H-3.17	W-1	W-1.49	W-1.99	W-2.49	W-3.59	C-2	C-3	C-4
5	0.25	0.59	0.90	0.35	0.71	1.06	0.44	0.67	0.92	1.06	1.51	1.28	1.81	2.36
10	0.83	1.49	2.08	1.08	1.79	2.54	1.30	1.82	2.31	2.66	3.59	3.46	4.79	6.11
15	1.70	2.62	3.47	2.16	3.25	4.41	2.59	3.41	4.21	4.78	6.39	6.92	9.50	12.31
20	2.82	3.95	5.01	3.48	4.97	6.57	4.23	5.28	6.42	7.36	9.75	11.02	14.63	19.86
25	4.18	5.45	6.65	5.11	6.77	8.70	5.98	7.14	8.47	9.55	12.43	14.26	18.39	25.24
30	5.72	7.13	8.45	6.84	8.66	10.88	7.84	9.18	10.66	11.84	15.04	17.34	21.93	29.85
35	7.45	8.91	10.25	8.72	10.65	13.01	9.81	11.25	12.87	14.21	17.70	20.23	25.13	34.21
40	9.35	10.87	12.22	10.75	12.73	15.18	11.91	13.42	15.05	16.44	20.12	22.87	27.95	37.69
45	11.43	12.97	14.31	12.92	14.93	17.41	14.15	15.69	17.37	18.80	22.52	25.40	30.73	40.77
50	13.69	15.20	16.49	15.29	17.27	19.78	16.56	18.13	19.80	21.26	25.01	28.03	33.65	43.87
55	16.11	17.54	18.76	17.83	19.74	22.23	19.11	20.68	22.36	23.84	27.59	30.76	36.43	47.09
60	18.68	20.06	21.15	20.51	22.35	24.84	21.81	23.33	25.05	26.52	30.28	33.71	39.40	50.37
65	21.37	22.72	23.68	23.33	25.10	27.53	24.60	26.17	27.91	29.38	33.08	36.52	42.60	53.79
70	24.27	25.51	26.30	26.35	28.17	30.36	27.61	29.14	30.85	32.36	36.61	40.93	46.26	57.29
75	27.34	28.44	29.02	29.28	31.16	33.25	30.86	32.27	33.97	35.32	-	43.08	49.70	60.91
80	30.54	31.54	31.90	32.47	34.36	36.23	-	35.79	-	-	-	46.40	52.23	64.59

#### 4.- Discussion.

Plots of partial or differential dilatation  $d(\text{mm}^3/\text{g}/5^\circ\text{C})$  against  $t_m(\text{ }^\circ\text{C})$  show the following general trend:

- a) All the curves have a pattern similar to that of water.
- b) The bigger solid content the bigger dilatation  $d$  in the zone from  $0^\circ$  to  $40^\circ\text{C}$ . Inversely, from  $40^\circ\text{C}$  upwards the differential dilatation decreases according to the bigger solid content; on the other hand, dilatations  $d$  in this zone exhibit a linear dependence with temperature.
- c) Melting effects are recorded as follows: Melting zone, corresponding to MMG (medium melting glycerides) with a maximum at about  $16^\circ\text{C}$ ; melting zone of HMG (high melting glycerides) with a maximum at about  $30^\circ\text{C}$ ; final melting point at about  $38^\circ\text{C}$ .
- d) In the milk cases, i.e., S, H or W milks, a temperature between  $40^\circ$  and  $60^\circ\text{C}$  is observed where all the samples belonging to the same milk type exhibit the same dilatation:  $42.5^\circ\text{C}$  for S serie,  $47.5^\circ\text{C}$  for H serie and  $57.5^\circ\text{C}$  for W serie.

The regression analysis of experimental data in the linear zone, i.e.,

$$d_1 = a + bt_m \quad \text{mm}^3/\text{g}/5^\circ\text{C}$$

give the following results:

T A B L E I-10

Regression parameters:  $d_1 = a + bt_m$  ( $\text{mm}^3/\text{g}/5^\circ\text{C}$ ) ,  $40^\circ < t < 80^\circ\text{C}$

Sample	a	b	r
S-1	0.6990	0.0325	0.9988
S'-1	0.7502	0.0317	0.9984
S-2.03	0.8872	0.0283	0.9992
S-3.46	1.1788	0.0212	0.9990
S-4.58	1.2150	0.0197	0.9966
H-1	0.8541	0.0316	0.9990
H-2.02	1.0343	0.0275	0.9986
H-3.17	1.2910	0.0226	0.9992
H-4.25	1.3055	0.0198	0.9876
W-1	0.9440	0.0307	0.9997
W-1.49	1.0692	0.0285	0.9978
W-1.99	1.1355	0.0275	0.9989
W-2.49	1.3007	0.0246	0.9980
W-3.59	1.4763	0.0215	0.9951
W-4.48	1.6477	0.0176	0.9930
C-2	1.5001	0.0240	0.9945
C-3	1.8618	0.0215	0.9954
C-4	2.2513	0.0185	0.9933